The Tokamak Density limit: a Thermoresistive Disruption Mechanism

Presented by D. A. Gates
In collaboration with
D.P.Brennan, L. Delgado-Aparicio, Q. Teng, R. B. White

At the 3rd Workshop for the Theory and Simulation of Disruptions

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Outline

- Review density limit scaling argument
- Issues with simple modification of the Modified Rutherford Equation (MRE)
- New physics in the MRE
- Results from the new model
- Summary

Where does the Greenwald limit come from?

- The empirical tokamak operational limit (aka the Greenwald limit) relates the maximum achievable density to the circular-equivalent current density

$$\bar{n}_e(10^{20}m^{-3}) < \frac{I_p}{\pi a^2}(MA/m^2)$$

  – A radiative limit should scale as $P^{1/2}$

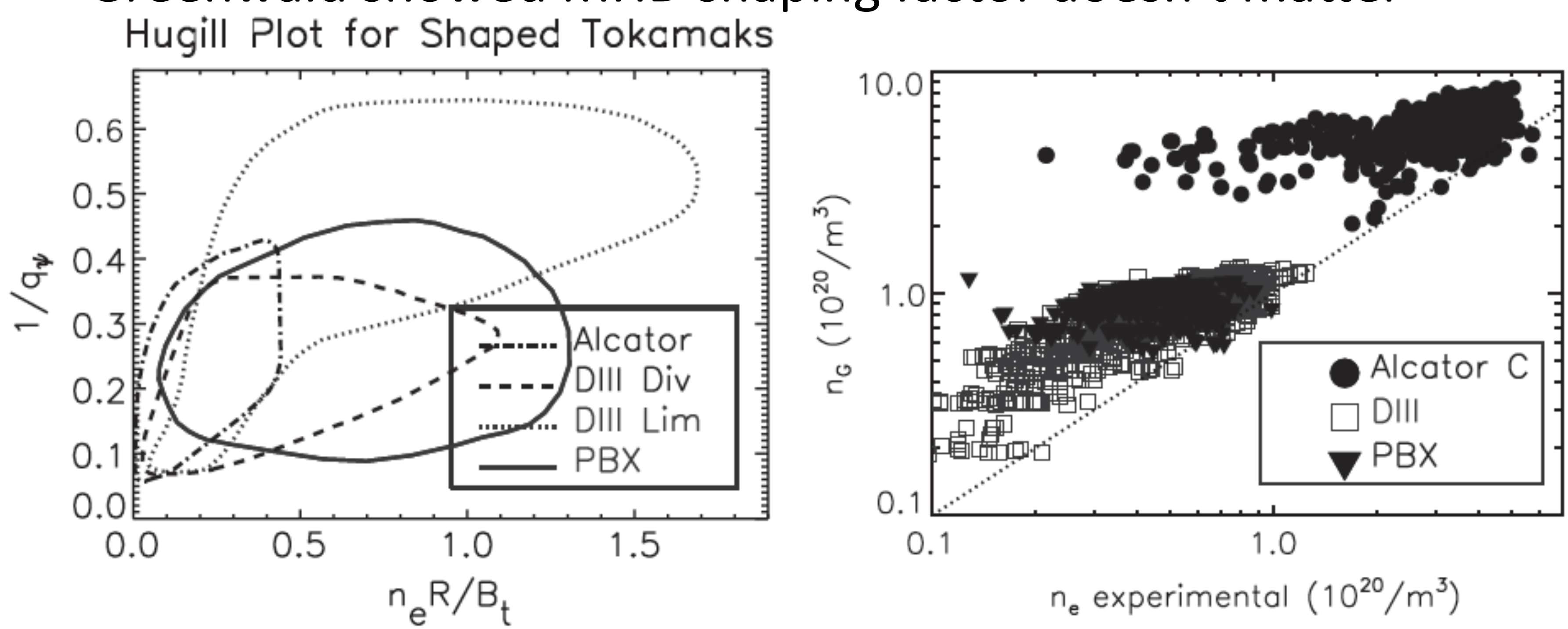
- The Greenwald limit is a fairly robust result

Puzzles associated with the Greenwald limit

1) The scaling is universal, but the phenomenon appears to be associated with radiative collapse and tearing modes, which can be complicated
2) If the physics is associated with radiative collapse, why is the density limit so weakly dependent on heating power?
3) Why is the limit only weakly dependent on Z_{eff}?
4) The collapse is associated with the onset of magnetic islands, so why does the limit not depend on plasma shaping or q (both which are known to affect MHD stability)?
5) Why is the density limit power scaling different in stellarators?
6) Why are tearing modes associated with a radiative collapse?

Evolution of the density limit

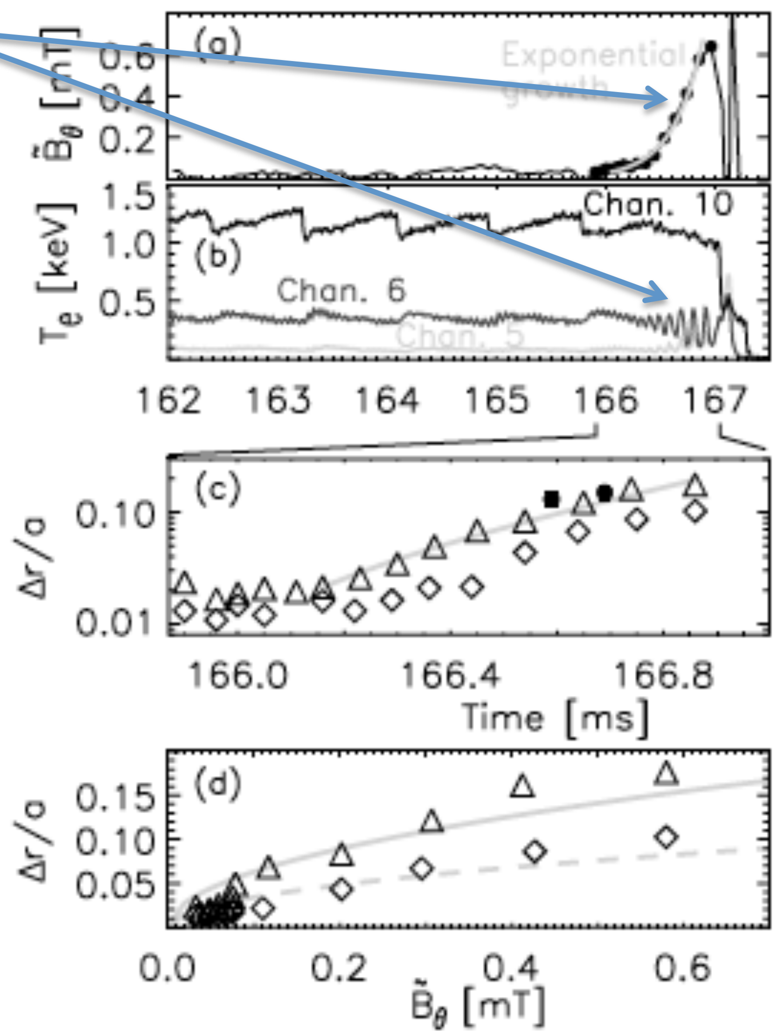
- The form of the density limit changed as databases from multiple tokamaks were amassed
- Hugill plot used q out of deference to MHD – works fine for circular cross-section machines
- Greenwald showed MHD shaping factor doesn't matter



M. Greenwald, et al., Nucl. Fusion, **28** (1988) 2199

Tearing modes precede the density limit collapse

- MHD mode preceding collapse is ubiquitous
- Explained by Wesson as a classical Δ' change caused by the l_i increase
  - Unfortunately, classical Δ' is a sensitive parameter -> not a robust effect
  - Never been successfully modeled from a stability point of view

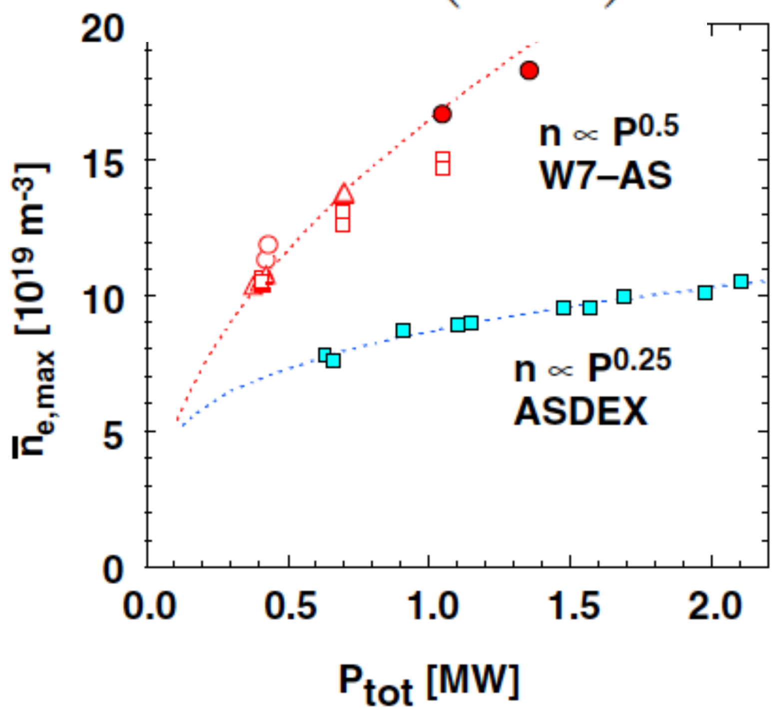


F. Salzedas, et al., PRL **88** (2002) 075002

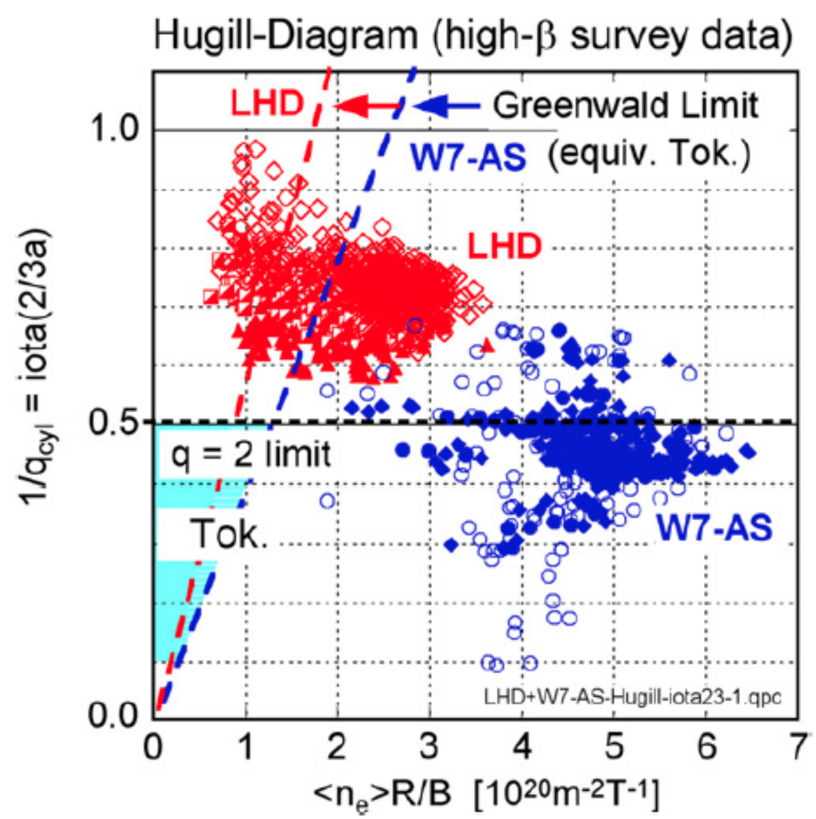
Stellarators are different than tokamaks

- Density limit clearly does not obey tokamak scalings
- Stellarator density limit is given by the Sudo limit

$$n_{\mathrm{lim}} = 0.25 \left(\frac{P_{\mathrm{in}} B_{\mathrm{T}}}{a^2 R} \right)^{0.5}$$



M. Greenwald, et al., Plasma Phys. Control. Fusion **44** (2002) R27–R80



A. Weller, et al., Nucl. Fusion **49** (2009) 065016

Radiation increases at the Greenwald limit

- Radiation physics matters!
  – Why doesn't the Greenwald limit depend on heating power?
- Collapse is not associated with fixed P_{rad}/P_{tot}

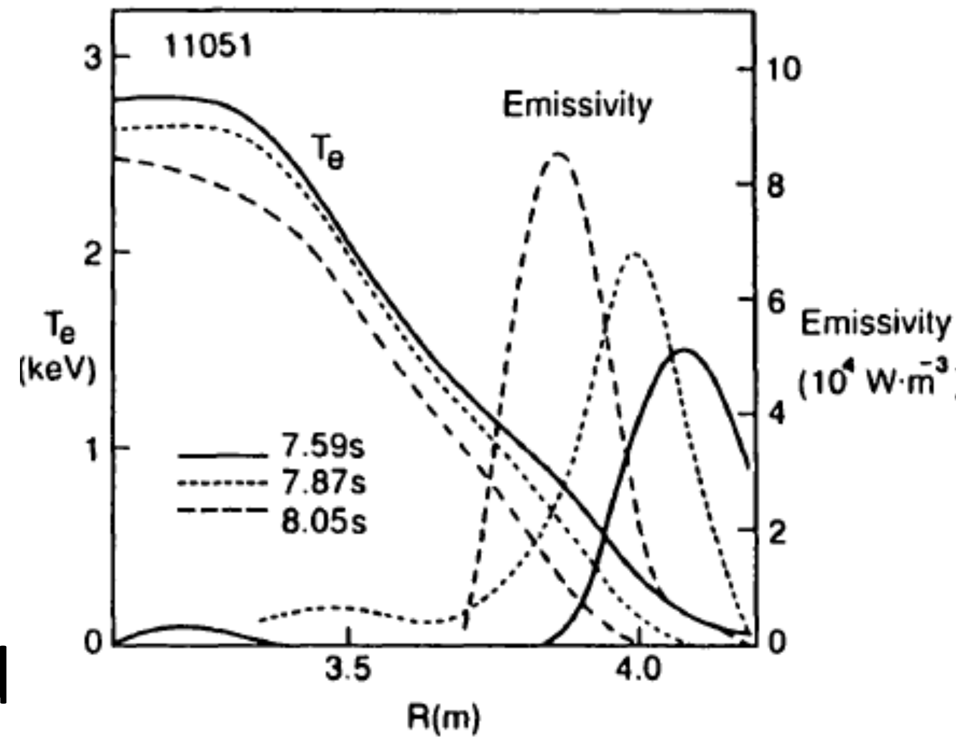


FIG. 12. Temperature and radiated power profiles during the plasma contraction.

*J. A. Wesson, R. D. Gill, M. Hugon, F. C. Schuller, J. A. Snipes, et al., Nucl. Fusion **29** (1989) 641

But density limit does not vary with Z_{eff}

- Density limit almost independent of Z_{eff} until Z_{eff}~3
- Z_{eff} is a good proxy for Brehmstrahlung

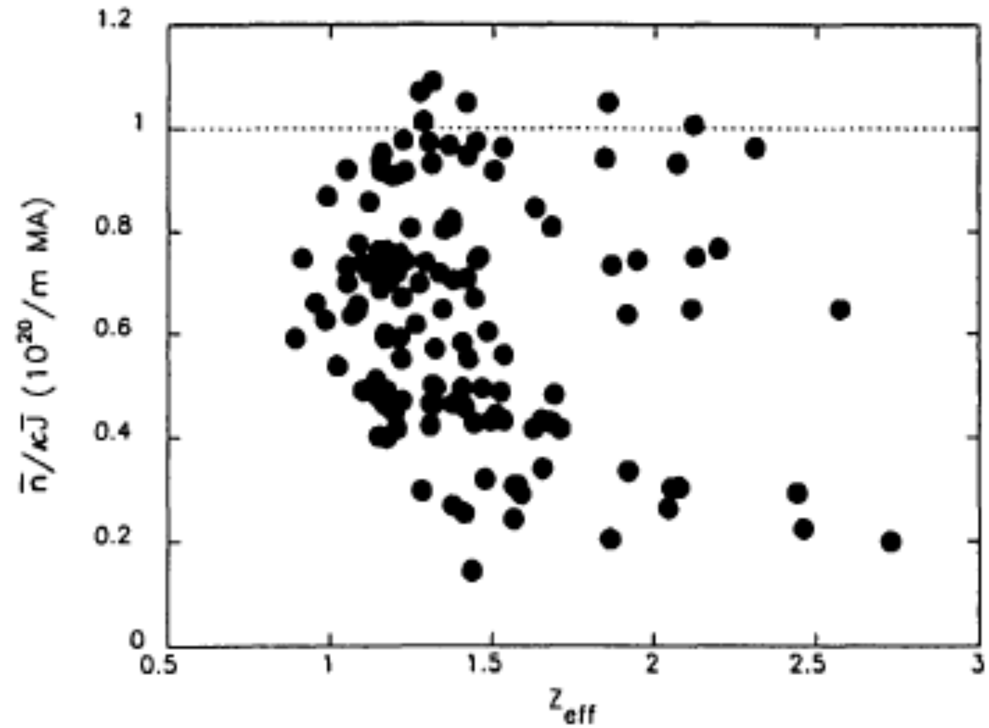


FIG. 9. Density (normalized to κJ) versus Z_{eff}. The dashed curve, which represents the scaled limit, $\bar{n} = \kappa\bar{J}$, can be reached for plasmas with Z_{eff} substantially above 1 (Alcator C).

M. Greenwald, et al., Nucl. Fusion, **28** (1988) 2199

Summary of issues

- Associated with radiation - but not heating power
  - Relatively insensitive to Z_{eff}
- Current matters (like MHD) but shape doesn't (not like MHD)
- MHD tearing modes occur
- An apparently complex phenomenon is universal

The islands at the density limit have been tentatively identified as radiation driven

- Suttrop et al. did extensive study on ASDEX-U (1997)
- Did not draw a causal connection between islands and the density limit
- Did identify at least some of the islands as potentially radiative

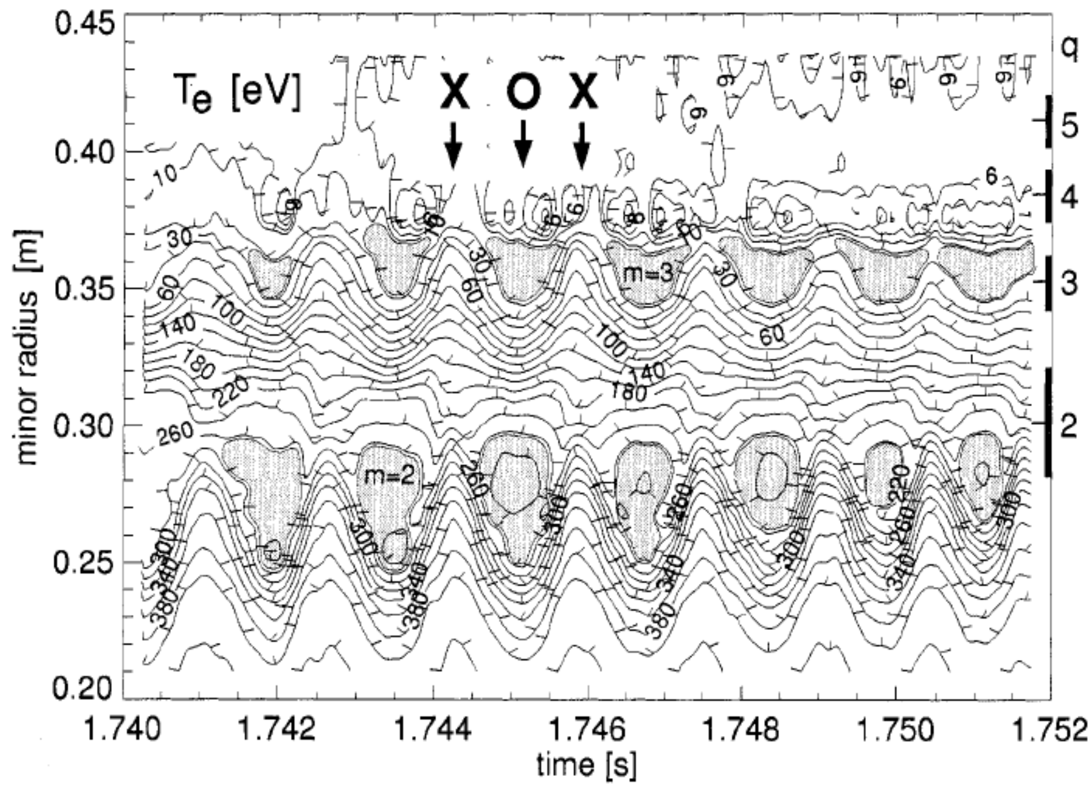
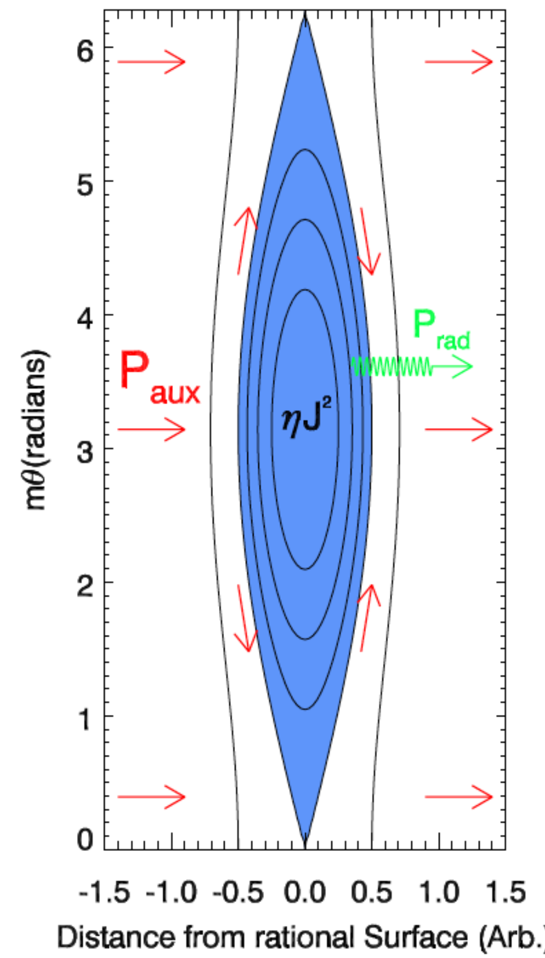


FIG. 2. Reconstruction of coupled (2,1) and (3,1) islands from T_e measurements in a time interval during current profile contraction between two minor disruptions. Islands recognized by regions of flat T_e are marked by shaded areas. While the (3,1) island grows, the (2,1) island shrinks. $q(r)$ is derived from equilibrium reconstruction at $t = 1.75$ s with radial uncertainties indicated.

W. Suttrop, et al., Nucl. Fusion, **37** (1997) 119

Radiation driven islands

- The island is magnetically insulated from it's surroundings
- So radiation can cool the island,
- Lower temperature leads to increased resistivity
- This enhances the helical current perturbation
- The island then grows, increasing the magnetic insulation, causing the process to exponentiate.



P. H. Rebut and M. Hugon, Plasma Physics and Controlled Nuclear Fusion Research 1984 (Proc. 10th Int. Conf. London, 1984), Vol. 2, IAEA, Vienna, 197, (1985).

Radiation drive in the MRE

- Power balance in the island

$$n_e \chi_\perp \nabla T_e A_{island} = \delta P * V_{island}$$

  – where A_{island} is the surface area of the integrated over the inside and outside of the island and V_{island} is the volume of the island.

- Relate the current to the temperature using resistivity and use Rutherford Δ' formula

$$\frac{\delta J}{J} = -\frac{3\delta T}{2T} \qquad \Delta' = 16 k_1 \frac{\delta J}{swJ}$$

- Find the radiation drive term

$$\Delta' = 3 \frac{r_s s_I}{s} \frac{\delta P}{n_e \chi_\perp T_e} w$$

P. H. Rebut and M. Hugon, Plasma Physics and Controlled Nuclear Fusion Research 1984 (Proc. 10th Int. Conf. London, 1984), Vol. 2, IAEA, Vienna, 197, (1985).

Modified Rutherford equation with radiation

$$\frac{k_0}{\eta}\frac{dw}{dt} = \Delta' r_s - C_1\left(\frac{w}{w^2 + w_\chi^2}\right) + \frac{C_2}{w^3} + C_3 w$$

Radiation term

Rutherford term Pressure driven current Polarization current

- For now, ignore the bootstrap and polarization terms (consider low to moderate β_p)
- The MRE then becomes:

$$\frac{k_0}{\eta}\frac{dw}{dt} = \Delta' r_s + C_3 w$$

Where: $C_3 = 3(r_s s_I / s)(\delta P/[n_e \chi_\perp T_e])$

Exponential growth

- The radiation term changes sign when δP = 0 or

$$P_{rad} = P_{heat} \longrightarrow P_{rad} \sim \eta J^2$$

Radiation drive term changes sign when island cools

$$P_{rad} < \eta J^2 \quad or \quad n_e E_{ave} \upsilon_{eZ} < \frac{m_e \upsilon_{ei}}{e^2 n_e} J^2$$

$$n_e < \sqrt{\frac{m_e}{e^2 E_{eff}} \frac{\upsilon_{ei}}{\upsilon_{eZ}}} J$$

- Assume ohmic heating dominates inside of the island
- Auxiliary power is shunted around the island by parallel conduction, consistent with density limit being independent of heating power
  - Constant temperature island boundary
- Quantity in square root is nearly independent of temperature*
- Reminiscent of the Greenwald limit

*F. W. Perkins and R. A. Hulse, Phys. Fluids **28** (1985) 1837.

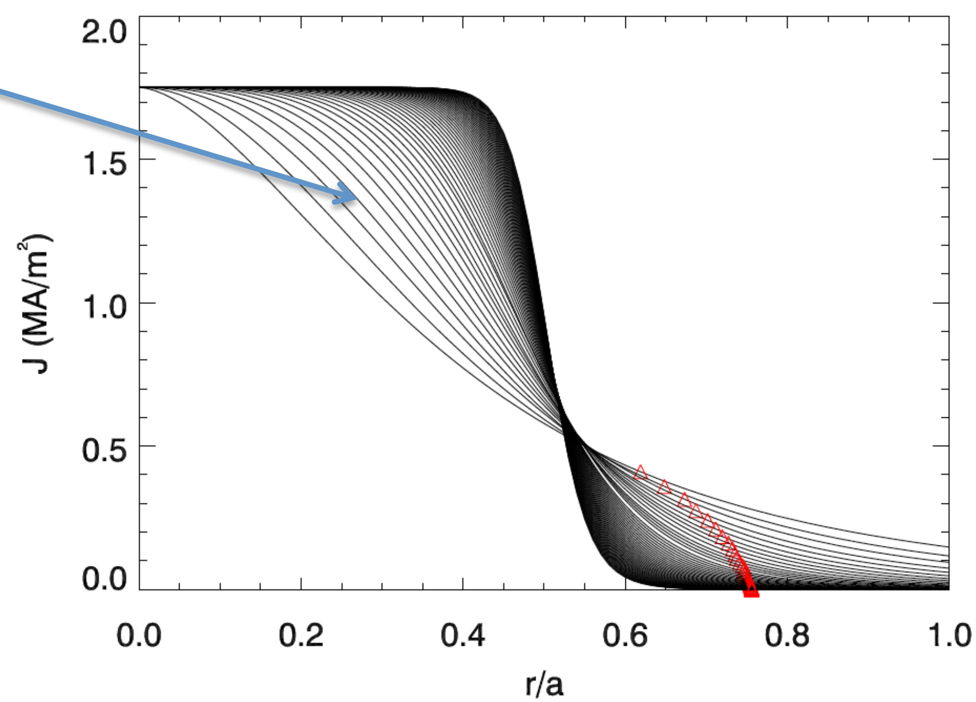
Simple cylindrical model relates local density and current to global values

- Use a simple profile model

$$J = \frac{J_0}{\left(1+\left(\frac{r}{r_0}\right)^{2\nu}\right)^{1+\frac{1}{\nu}}}$$

- Assume parabolic density profile
- Still to many variables
  - Need additional information to determine J(r) at the density limit

Current profiles used in simple density limit model at constant-q



Current profile peaking at the density limit

- Corresponding to the density limit there is a corresponding (simultaneous) l_i-limit
- Fit this curve with a line

$$l_i = 0.12 q_{edge} + 0.6$$

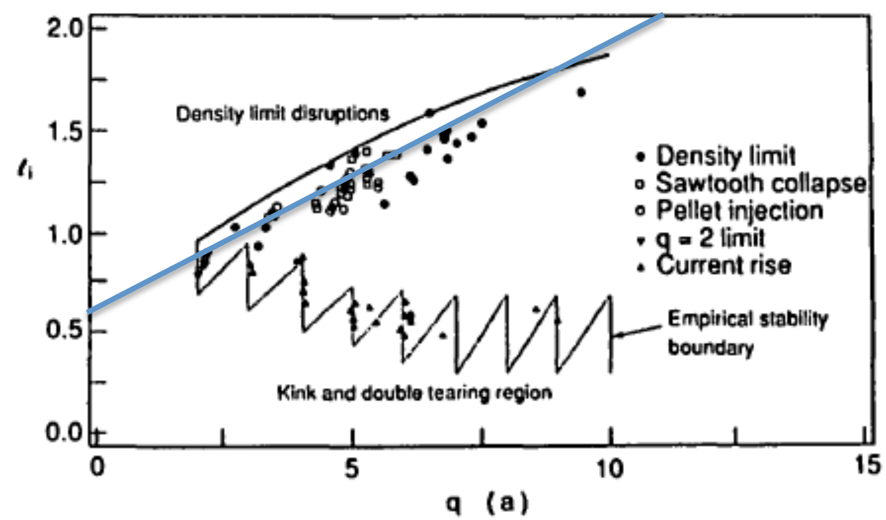
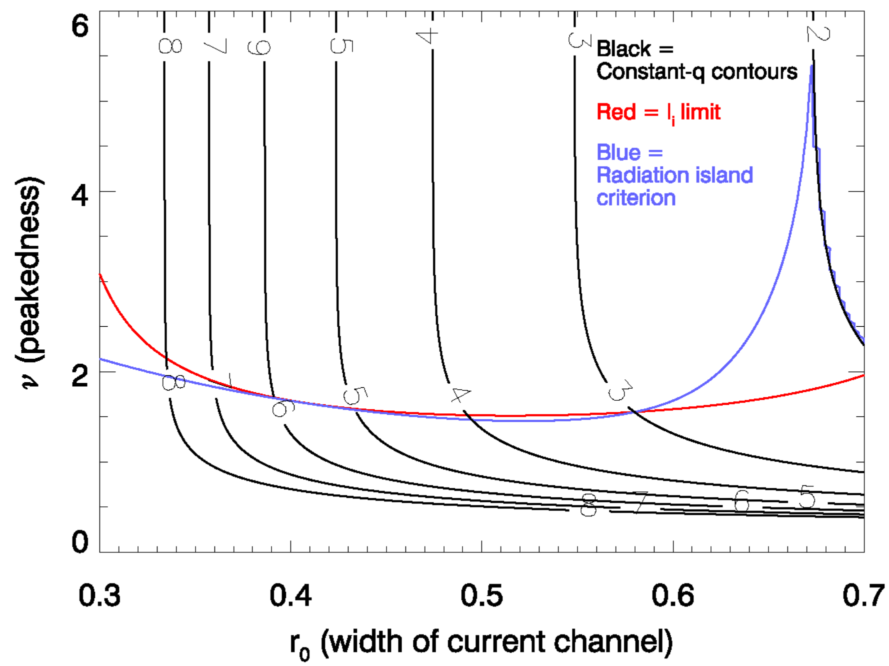


FIG. 6. Empirical stability diagram for JET, showing the l_i–q plane. The lower boundary (dotted) indicates the stability boundary for rotating MHD modes during the current rise. The upper boundary (solid) indicates the region where major disruptions occur. The symbols indicate the onset of quasi-stationary modes in various situations.

*J. A. Wesson, R. D. Gill, M. Hugon, F. C. Schuller, J. A. Snipes, et al., Nucl. Fusion **29** (1989) 641

A contour of constant local power balance correspond with the contour of maximum l_i

- Indicates that the local and the global scaling laws are co-linear if the current profile corresponds with the l_i observed at the density limit
  - ➢ Island net power threshold corresponds to the Greenwald limit



Contour plot of the 1) q_{edge} (black) as a function of the profile parameters ν and r_0. Also shown in the plot are 2) l_i (red) and the 3) island net power threshold (blue)

Model doesn't reproduce observations

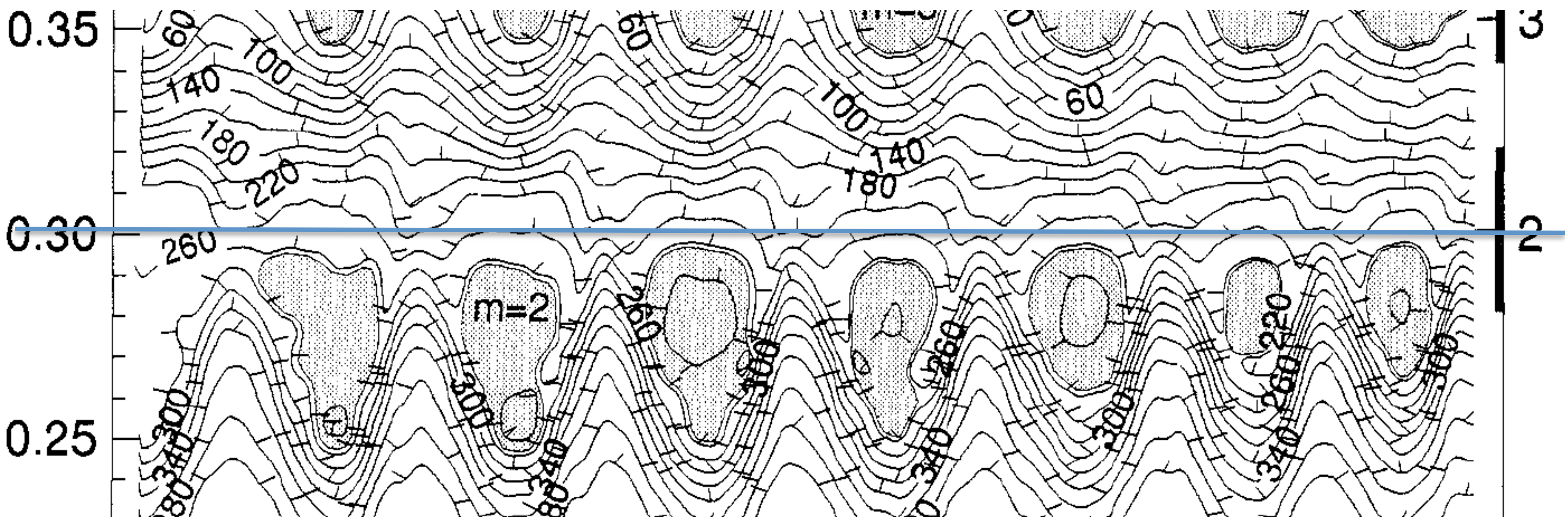
$$\frac{k_0}{\eta}\frac{dw}{dt} = \Delta'(w)r_s + C_3 w$$

$$\Delta'(w)r_s = (\Delta' - \alpha w)r_s$$

- White saturation term becomes stabilizing at large width
- δT required to drive a large island is too big
  - (Hegna, private communication)
- Large temperature depression in the center of islands is not observed in experiments.
- Something else must be going on…

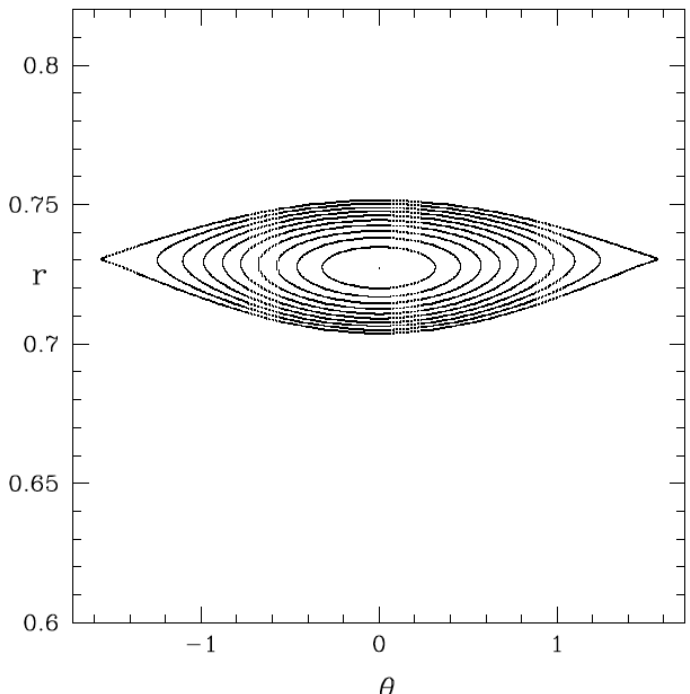
Observed islands are asymmetric

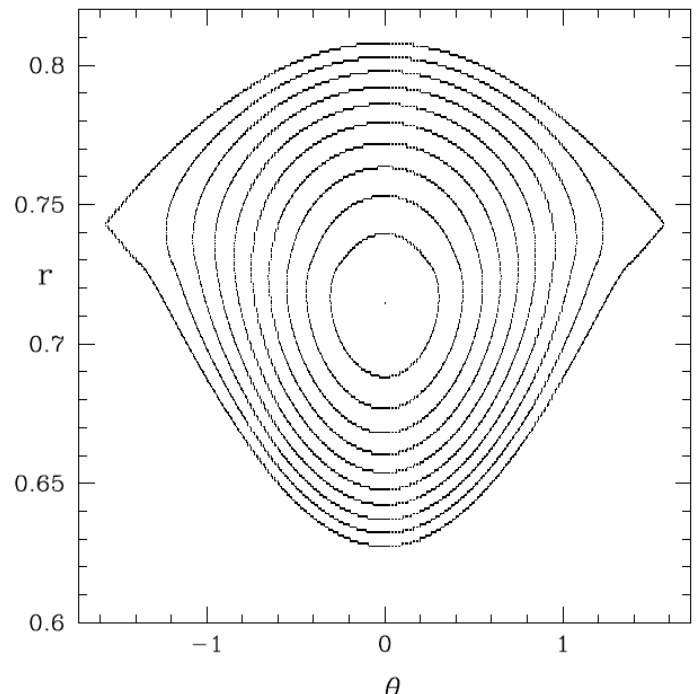
- O-point of the 2/1 island is shifted 3cm inside the nominal q=2 surface
- X-point appears shifted outside of q=2 surface
- Inward shift of the island is destabilizing for flattened islands



Island from linear theory is asymmetric at large width

- Asymmetry at large width is not an additional non-linearity– well established
- Was previously considered with constant η
  - R. B. White, D. A. Monticello, M. N. Rosenbluth, and B. V. Waddell, Phys. Fluids **20** , 800 (1977).





R. B. White, D. A. Gates, D. P. Brennan, Phys. Plasmas **22**, 022514 (2015)

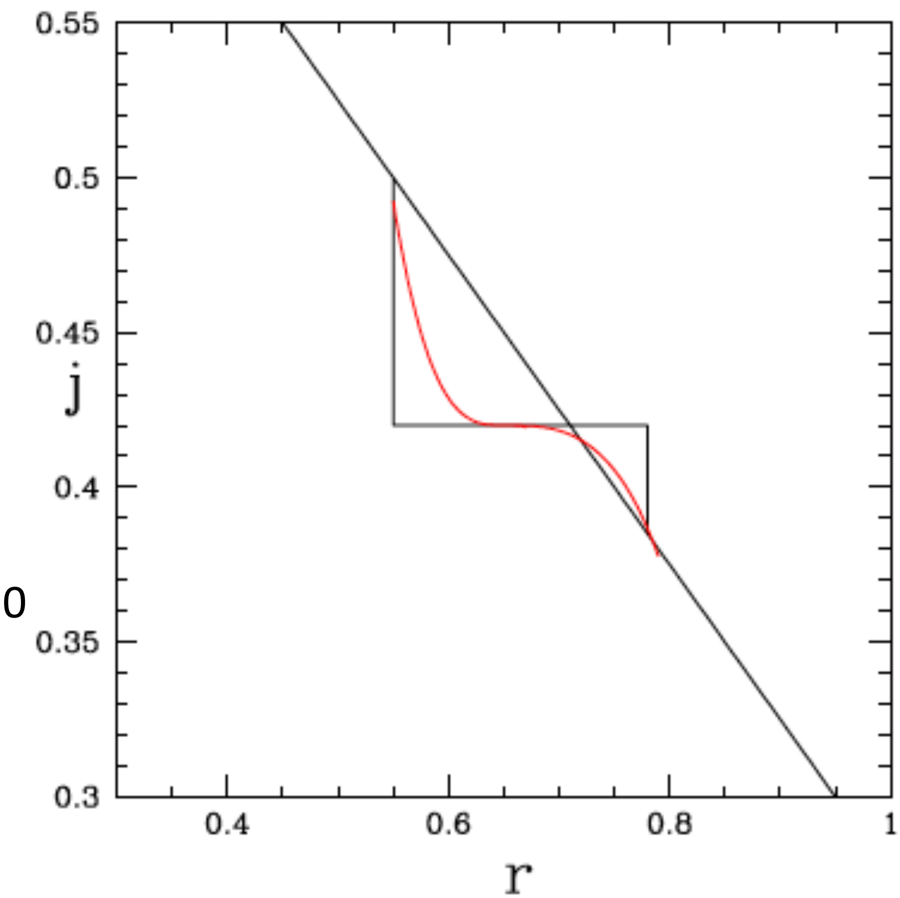
3D resistivity with asymmetry adds new destabilizing term

- Asymmetric resistivity perturbation with flattening leads to net negative current perturbation, so island grows

$$\Delta'_A(w) = f_F \frac{\int [j(r_x) - j(r)]dr}{\psi_1(r_s)}$$

E.Westerhof, et al., Nucl. Fusion **47** (2007) 85-90

- Must also include "Fitzpatrick effect"



New model with 3D resistivity

- Including all terms with 3D resistivity

$$\frac{dw}{dt} = r_s^2[\Delta'(w) + \Delta'_{\delta j}(w) + \Delta'_A(w)]$$

Rebut term

Rutherford/White term

Asymmetry (Westerhof) term

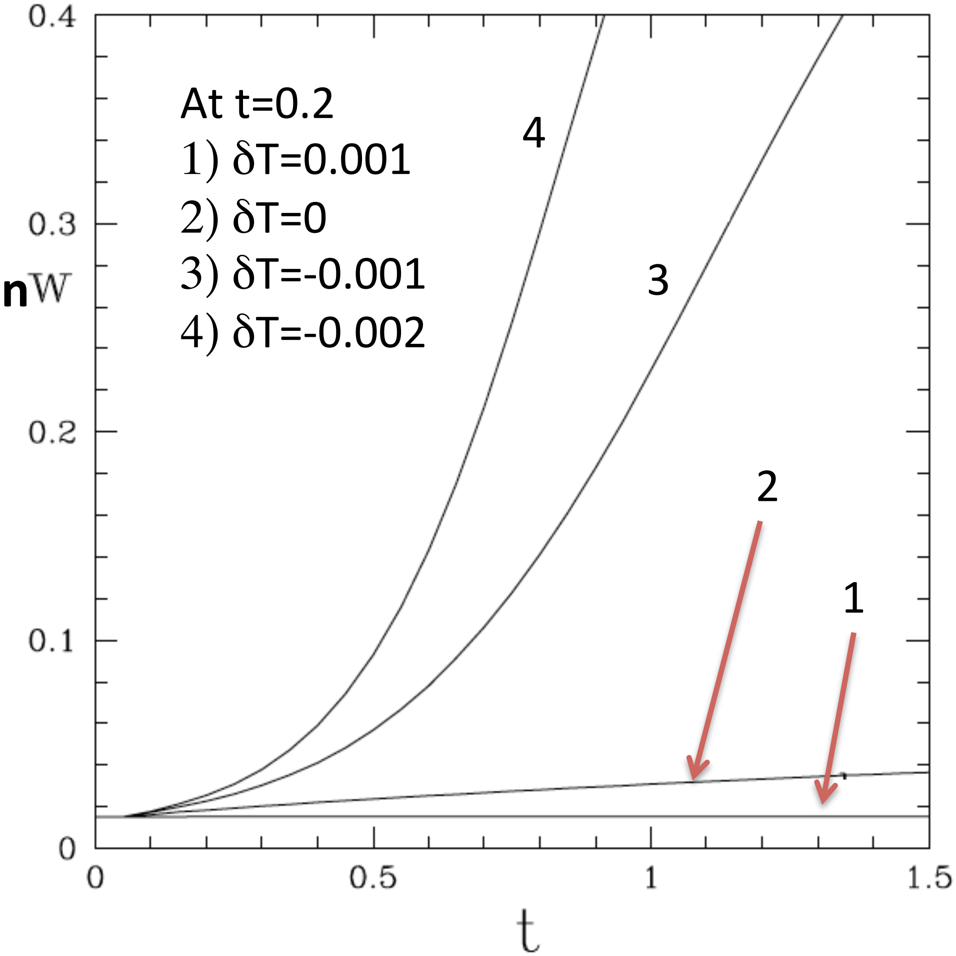
$$\Delta'_{\delta j}(w) = -\frac{32}{\pi}\frac{\delta j_1}{\psi_0''}\frac{w}{w^2 + w_F^2}, \qquad \Delta'_A(w) = \frac{8 j'(r_s)}{\pi \psi_0''}\frac{w^2}{w^2 + w_F^2} f_A,$$

Fitzpatrick term

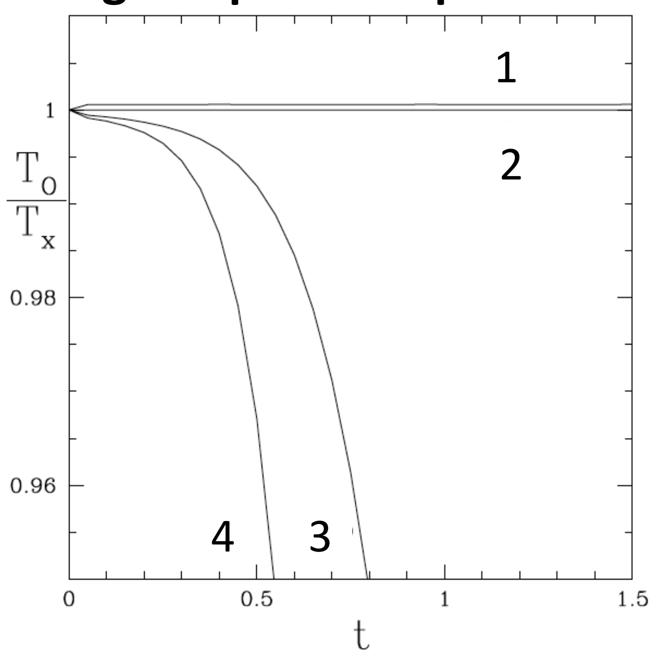
New model shows robust threshold for mode onset at identified density limit criteria

- Small change in internal temperature leads to rapid change in growth rate
- Resultant island has small temperature perturbation at the O-point

Time history of island width for different fixed internal temperature gradients



Corresponding temperature perturbation



Basis for the nonlinear MHD simulations, DEBS: Nonlinear full-MHD in cylindrical geometry

DEBS advances the vector potential, velocity and pressure

D. D. Schnack et al. Comp. Phys. Comm. 43, 17 (1986).

Simulation parameters and details

3D resistivity $\eta(r,\theta,z) \sim T^{3/2}$

Very low $\beta_0 = 0.005$

Thermal anisotropy (fixed) $\kappa_{\parallel}/\kappa_{\perp} = 10^5$

Semi-implicit normalized to $\tau_R = \dfrac{4\pi a^2}{c^2\eta_0}$

$S_0 = \tau_R/\tau_A = 10^6$

$P = \nu/\eta_0 = 0.1$

$a/R = 0.5$

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{V} \times \mathbf{B} - \frac{\eta}{S_0}\mathbf{J}$$

$$\frac{\partial \mathbf{V}}{\partial t} = -\mathbf{V}\cdot\nabla\mathbf{V} + \frac{1}{\rho}\mathbf{J}\times\mathbf{B} - \frac{\beta_0}{2\rho}\nabla p + \frac{\nu}{\rho}\nabla^2\mathbf{V}$$

$$\frac{dp}{dt} = -\nabla\cdot(p\mathbf{V}) - (\gamma - 1)p\nabla\cdot\mathbf{V} + \frac{1}{S_0}\nabla\cdot(\kappa_{\perp}\nabla_{\perp}T + \kappa_{\parallel}\nabla_{\parallel}T)$$

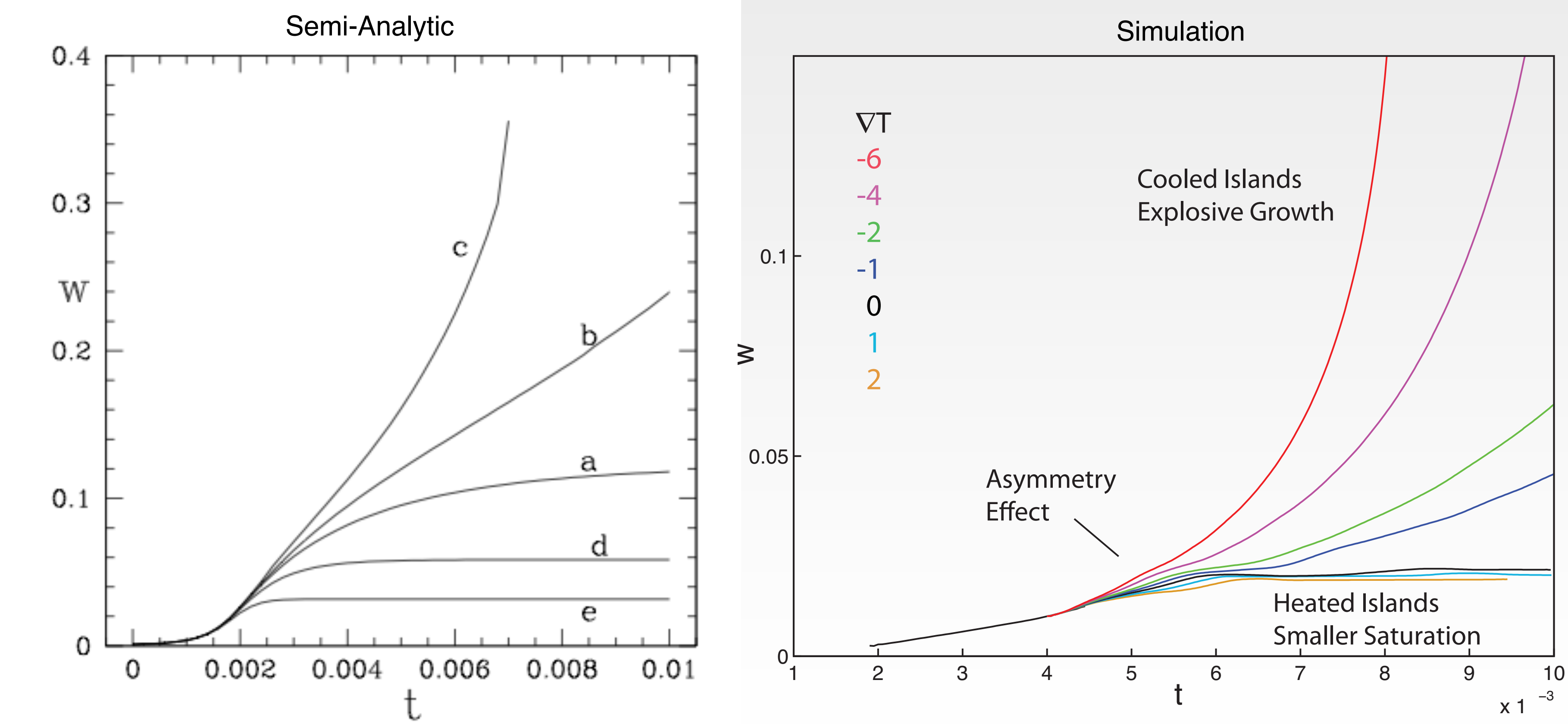
where

$\nabla \times \mathbf{A} = \mathbf{B}$

$\nabla \times \mathbf{B} = \mathbf{J}$

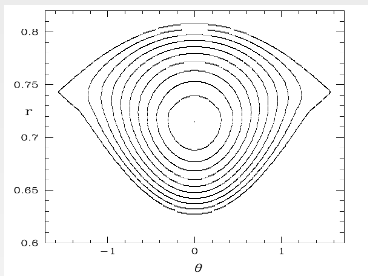
Simulations agree with reduced analysis, cooling causes exponential growth, heating causes saturation

Islands with small amount of cooling eventually exponentially grow in w
Temperature perturbations are small, mostly below experimental observation
Despite tiny delta-T, heated islands saturate at small size ($\eta(T)$ has strong effect)

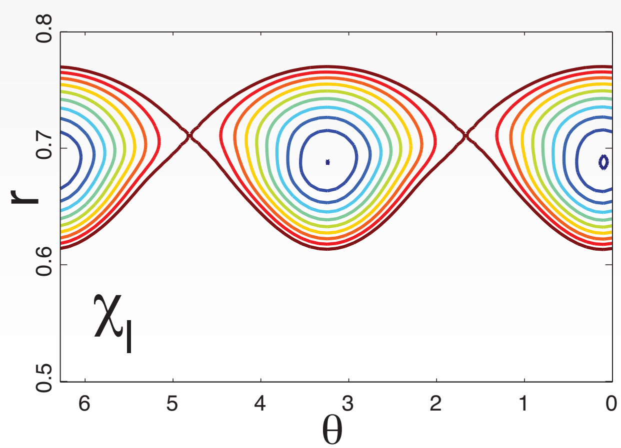


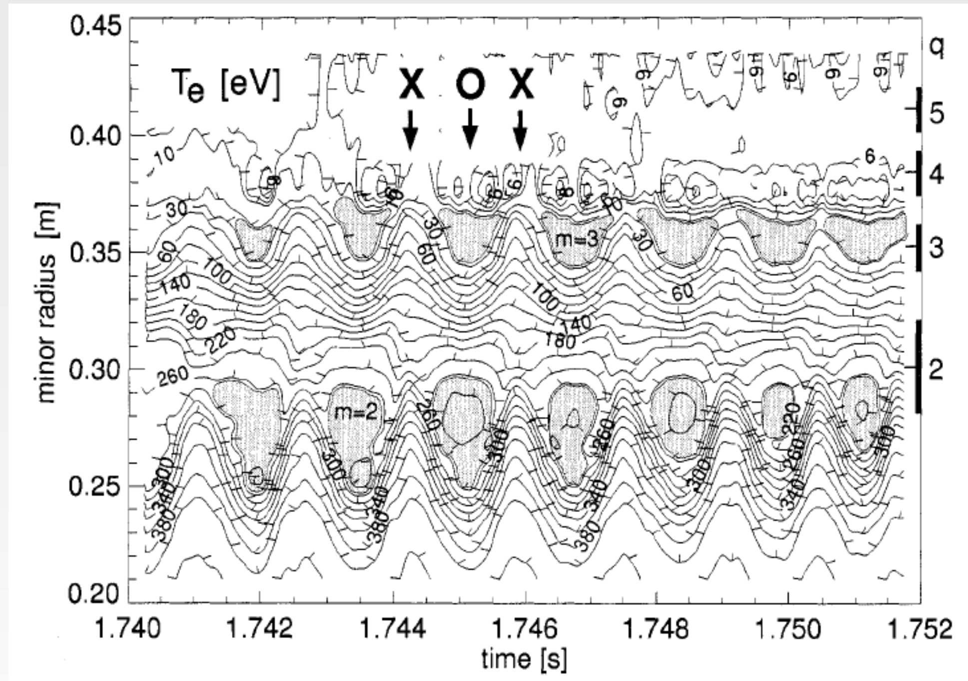
Asymmetric structure and 3D resistivity are key to capturing this physics, ignored in past

White et al: Analytic analysis of asymmetric island structure



Brennan: Next talk



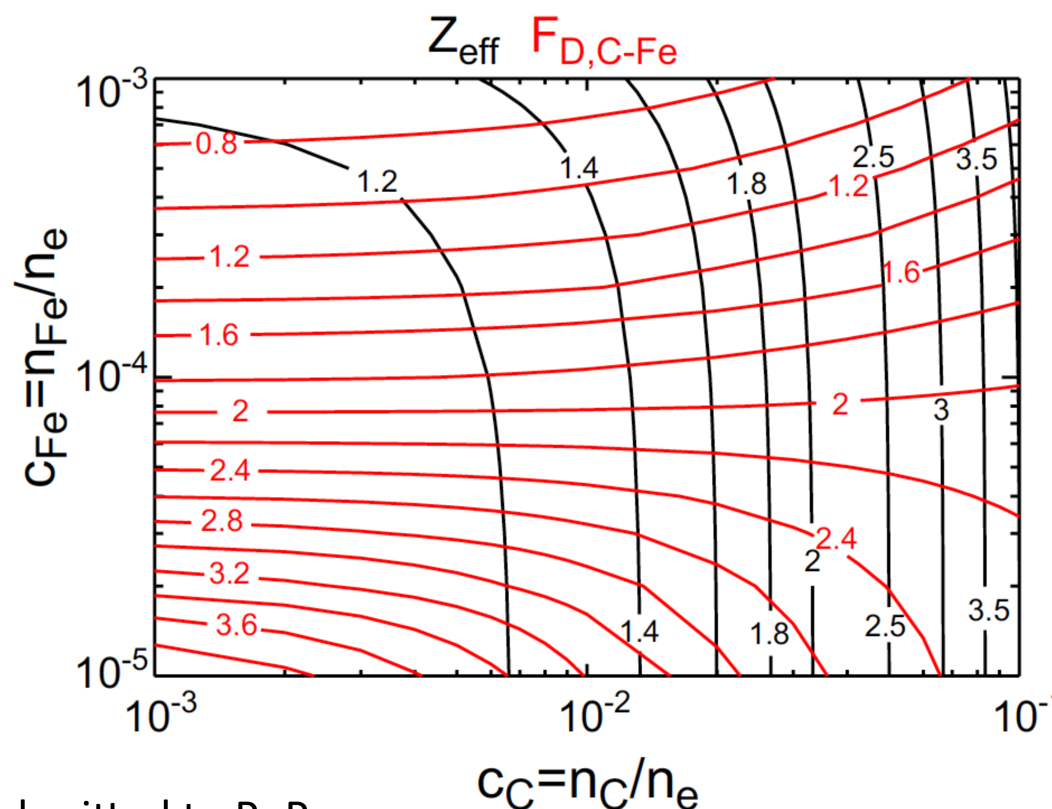


Suttrop et al: Experimental observation (ECE) of asymmetric islands preceding a density limit disruption.

Density limit coefficient is nearly independent of Z_{eff}

- Local coefficient between current and density depends predominantly on line radiation from high-Z impurities
  - Z_{eff} depends predominantly on low-Z

Contours of coefficient between density and current along with contours of constant Zeff for a three species plasma.



L. Delgado-Aparicio, D. A. Gates, to be submitted to PoP
D.E. Post, et al., At. Data Nucl. Data Tables **20**, 397 (1977).

Simple low-beta tokamak model

- Assume 1% C and 1.1*x*10^{-4} Fe
- Assume constant E = j/η
- Assume ad hoc model for relationship between li and ne that is consistent with published data at the the density limit

$$l_i = \begin{cases} 0.12 \times q_{edge} \times \frac{n_e}{n_G} + 0.6 & \text{if } n_e/n_G > 0.7 \\ 0.084 \times q_{edge} + 0.6 & \text{if } n_e/n_G \leq 0.7 \end{cases}$$

- Use definitions:

$$P_{heat} = \eta j^2$$

$$\eta = 1.03 \times 10^{-4} Z ln\Lambda T[eV]^{-3/2} \Omega m$$

$$P_{rad} = n_e n_D L_D(T_e) + \sum_Z n_e n_Z L_Z(T_e)$$

L_Z defined in D.E. Post, et al., At. Data Nucl. Data Tables **20**, 397 (1977).

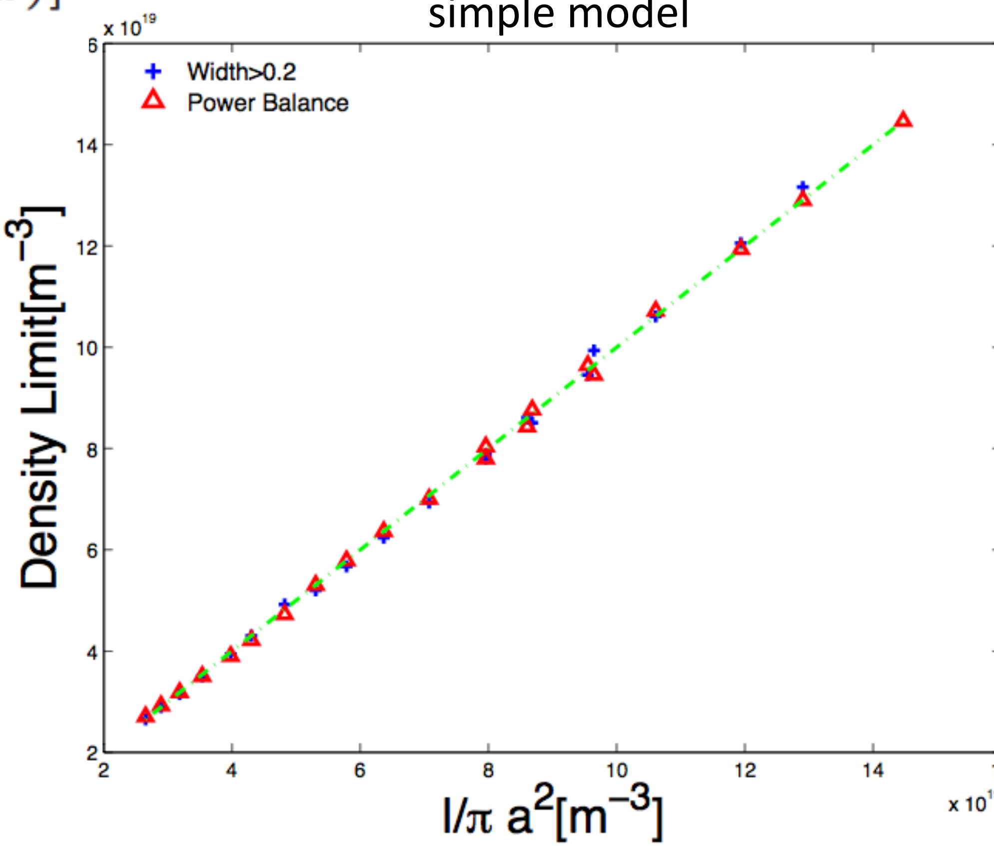
Model regenerates the Greenwald limit quantitatively

- Use model to solve

$$\frac{dw}{dt} = r_s^2[\Delta'(w) + \Delta'_{\delta j}(w) + \Delta'_A(w)]$$

  as a function n_e and I_p
- Reasonable assumptions and simple model give quantitative agreement with:
  - Scaling of the density limit
  - Sudden mode onset
  - Z_{eff} independence

Mode onset and power balance threshold from simple model



Q. Teng Bulletin of the APS (2015)

Implications and future plans

- This theory provides a testable quantitative prediction of the density limit based on local measurements and points to methods for exceeding the limit and controlling/avoiding disruptions
  - Key issue for ITER
- Theory predicts exponentially growing islands with a sudden robust onset condition
  - Consistent with a robust density limit and observed rabidly growing 2/1 tearing mode
  - Quantitatively predicts Greenwald limit with reasonable assumptions and simple model of a tokamak
- Need to directly verify local power balance criteria
  - Data analysis proceeding on NSTX
  - Experiments proposed on DIII-D, EAST, KSTAR
- Theory is robust – may be more widely applicable